Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Subsidiary

Monday 19 January 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

H31013A

1. A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

2.

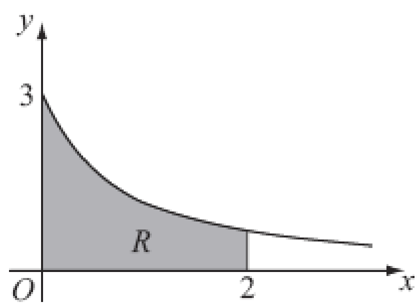

Figure 1

Figure 1 shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in Figure 1.

(a) Use integration to find the area of R.

(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)

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3. $$\mathrm{f}(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}.$$

Given that f(x) can be expressed in the form

$$\mathrm{f}(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^2. Simplify each term.

(6)

(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of f(0.2). Give your answer to 2 significant figures.

(4)

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4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

5.

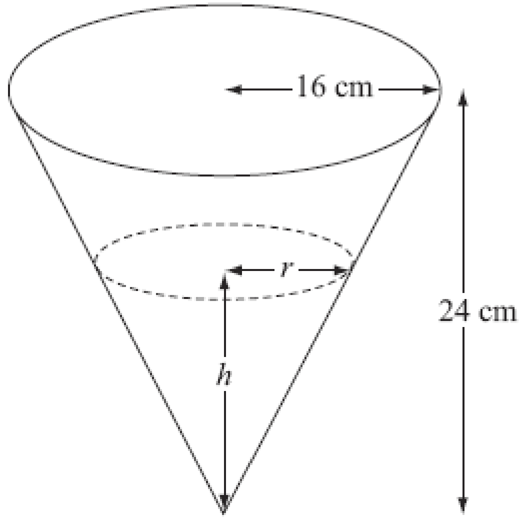

Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(a) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \dfrac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(b) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

6. (*a*) Find $\int \tan^2 x \ \mathrm{d}x$.

(2)

(*b*) Use integration by parts to find $\int \frac{1}{x^3} \ln x \ \mathrm{d}x$.

(4)

(*c*) Use the substitution $u = 1 + \mathrm{e}^x$ to show that

$$\int \frac{\mathrm{e}^{3x}}{1+\mathrm{e}^x} \ \mathrm{d}x = \frac{1}{2}\mathrm{e}^{2x} - \mathrm{e}^x + \ln(1 + \mathrm{e}^x) + k,$$

where k is a constant.

(7)

7.

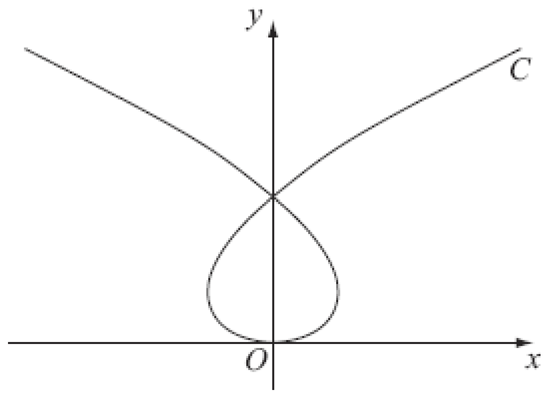

Figure 3

The curve C shown in Figure 3 has parametric equations

$$x = t^3 - 8t, \quad y = t^2$$

where t is a parameter. Given that the point A has parameter $t = -1$,

(a) find the coordinates of A.

(1)

The line l is the tangent to C at A.

(b) Show that an equation for l is $2x - 5y - 9 = 0$.

(5)

The line l also intersects the curve at the point B.

(c) Find the coordinates of B.

(6)

TOTAL FOR PAPER: 75 MARKS

END

January 2009
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1 (a)	C: $y^2 - 3y = x^3 + 8$		
	$\left\{\frac{\cancel{dy}}{\cancel{dx}} \cancel{\times}\right\}$ $2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{dy}{dx} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{dy}{dx} - 3\frac{dy}{dx}$'. Can be implied.	M1
	$\frac{dy}{dx} = \frac{3x^2}{2y-3}$	$\underline{\frac{3x^2}{2y-3}}$	A1 oe
			(4)
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2,3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$	$\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			(3)
	1(b) final A1 $\sqrt{}$ **.** Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, ***may*** indicate a correct follow through.		
			[7]

Question Number	Scheme		Marks	
2 (a)	Area(R) = $\int_0^2 \frac{3}{\sqrt{(1+4x)}}\,dx = \int_0^2 3(1+4x)^{-\frac{1}{2}}\,dx$			
	$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4}\right]_0^2$	***Integrating*** $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.	M1	
		Correct integration. Ignore limits.	A1	
	$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}}\right]_0^2$			
	$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$	Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.	M1	
	$= \frac{9}{2} - \frac{3}{2} = \underline{3}$ (units)^2	$\underline{3}$	A1	
				(4)
	(Answer of 3 with no working scores M0A0M0A0.)			
(b)	Volume $= \pi \int_0^2 \left(\frac{3}{\sqrt{(1+4x)}}\right)^2 dx$	Use of $V = \pi \int y^2\, dx$. Can be implied. Ignore limits and dx.	B1	
	$= (\pi) \int_0^2 \frac{9}{1+4x}\,dx$			
	$= (\pi)\left[\frac{9}{4}\ln\lvert 1+4x\rvert\right]_0^2$	$\pm k \ln\lvert 1+4x\rvert$	M1	
		$\frac{9}{4}\ln\lvert 1+4x\rvert$	A1	
	$= (\pi)\left[\left(\frac{9}{4}\ln 9\right) - \left(\frac{9}{4}\ln 1\right)\right]$ Note that $\ln 1$ can be implied as equal to 0.	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1	
	So Volume $= \underline{\frac{9}{4}\pi \ln 9}$ Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.	$\underline{\frac{9}{4}\pi\ln 9}$ or $\underline{\frac{9}{2}\pi\ln 3}$ or $\underline{\frac{18}{4}\pi\ln 3}$ Note that $= \frac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.	A1 oe isw	(5)
				[9]

Question Number	Scheme		Marks
3 (a)	$27x^2+32x+16 \equiv A(3x+2)(1-x) + B(1-x) + C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3},\quad 12-\frac{64}{3}+16 = \left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3} = \left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1,\quad 27+32+16 = 25C \Rightarrow 75 = 25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A \Rightarrow A=0$ $x=0,\quad 16 = 2A+B+4C \Rightarrow 16 = 2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
			(4)
(b)	$f(x) = \frac{4}{(3x+2)^2} + \frac{3}{(1-x)}$		
	$= 4(3x+2)^{-2} + 3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$= 4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right] + 3(1-x)^{-1}$		
	$= 1\left(1+\frac{3}{2}x\right)^{-2} + 3(1-x)^{-1}$		
	$= 1\left\{\underline{1+(-2)(\tfrac{3x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{3x}{2})^2+\ldots}\right\}$	Either $1\pm(-2)(\frac{3x}{2})$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+\,3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$	Ignoring 1 and 3, any one correct $\{\underline{\ldots\ldots\ldots}\}$ expansion.	A1
		Both $\{\underline{\ldots\ldots\ldots}\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$		
	$=4+0x\,;+\frac{39}{4}x^2$	$4+(0x)\,;\ \frac{39}{4}x^2$	A1; A1
			(6)

Question Number	Scheme		Marks
(c)	$\text{Actual} = \text{f}(0.2) = \dfrac{1.08+6.4+16}{(6.76)(0.8)}$ $= \dfrac{23.48}{5.408} = 4.341715976... = \dfrac{2935}{676}$	Attempt to find the actual value of f(0.2) or seeing awrt 4.3 and believing it is candidate's actual f(0.2).	
	Or $\text{Actual} = \text{f}(0.2) = \dfrac{4}{(3(0.2)+2)^2} + \dfrac{3}{(1-0.2)}$ $= \dfrac{4}{6.76} + 3.75 = 4.341715976... = \dfrac{2935}{676}$	Candidates can also attempt to find the actual value by using $\dfrac{A}{(3x+2)} + \dfrac{B}{(3x+2)^2} + \dfrac{C}{(1-x)}$ with their A, B and C.	M1
	$\text{Estimate} = \text{f}(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$	Attempt to find an estimate for f(0.2) using their answer to (b)	M1 $\sqrt{}$
	$\%\text{age error} = \dfrac{\lvert 4.39 - 4.341715976... \rvert}{4.341715976...} \times 100$	$\left\lvert \dfrac{\text{their estimate - actual}}{\text{actual}} \right\rvert \times 100$	M1
	$= 1.112095408... = 1.1\%(2\text{sf})$	1.1%	A1 **cao** (4) **[14]**

Question Number	Scheme		Marks
4 (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 cso
			(2)
(b)	Lines meet where:		
	$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$		
	First two of $\begin{array}{llll} \mathbf{i}: & 11 - 2\lambda & = -5 + q\mu & (1) \\ \mathbf{j}: & 2 + \lambda & = 11 + 2\mu & (2) \\ \mathbf{k}: & 17 - 4\lambda & = p + 2\mu & (3) \end{array}$	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
		Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their $\mathbf{k}$ component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 cso
			(6)
(c)	$\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2.	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			(2)

Question Number	Scheme		Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1 $\sqrt{}$ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$	$\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} \text{their } \overrightarrow{AX} \end{pmatrix}$	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			(3)
			[13]

Question Number	Scheme	Marks
5 (a)	Similar triangles $\Rightarrow \underline{\dfrac{r}{h}=\dfrac{16}{24}} \Rightarrow \underline{r=\dfrac{2h}{3}}$ — Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V=\dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi\left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG** — Substitutes $r=\frac{2h}{3}$ into the formula for the volume of water V.	A1
		(2)
(b)	From the question, $\dfrac{dV}{dt}=8$ — $\dfrac{dV}{dt}=8$	B1
	$\underline{\dfrac{dV}{dh}}=\underline{\dfrac{12\pi h^2}{27}}=\underline{\dfrac{4\pi h^2}{9}}$ — $\underline{\dfrac{dV}{dh}}=\underline{\dfrac{12\pi h^2}{27}}$ or $\underline{\dfrac{4\pi h^2}{9}}$	B1
	$\dfrac{dh}{dt}=\dfrac{dV}{dt}\div\dfrac{dV}{dh} = \underline{8\times\dfrac{9}{4\pi h^2}} = \underline{\dfrac{18}{\pi h^2}}$ — Candidate's $\dfrac{dV}{dt}\div\dfrac{dV}{dh}$;	M1;
	$\underline{8\div\left(\dfrac{12\pi h^2}{27}\right)}$ or $\underline{8\times\dfrac{9}{4\pi h^2}}$ or $\underline{\dfrac{18}{\pi h^2}}$ oe	A1
	When $h=12$, $\dfrac{dh}{dt}=\underline{\dfrac{18}{144\pi}}=\underline{\dfrac{1}{8\pi}}$ — $\underline{\dfrac{18}{144\pi}}$ or $\underline{\dfrac{1}{8\pi}}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\underline{\dfrac{18}{144\pi}}$.	(5)
		[7]

Question Number	Scheme		Marks
6 (a)	$\int \tan^2 x \, \mathrm{d}x$		
	$\left[NB: \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1}\right]$	The correct <u>underlined identity</u>.	M1 oe
	$= \int \sec^2 x - 1 \, \mathrm{d}x$		
	$= \underline{\tan x - x} \, (+c)$	Correct integration with/without + c	A1
			(2)
(b)	$\int \frac{1}{x^3} \ln x \, \mathrm{d}x$		
	$\left\{ \begin{array}{l} u = \ln x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^{-3} \quad \Rightarrow \quad v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$		
	$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, \mathrm{d}x$	Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$.	M1
		Correct expression.	A1
	$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, \mathrm{d}x$	An attempt to multiply through $\frac{k}{x^n}, n \in \square, n \ldots 2$ by $\frac{1}{x}$ and an attempt to ...	
	$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)} \, (+c)$	... "integrate"(process the result);	M1
		<u>correct solution</u> with/without + c	A1 oe
			(4)

Question Number	Scheme		Marks
(c)	$\int \frac{e^{3x}}{1+e^x}\,dx$		
	$\left\{u = 1+e^x \Rightarrow \underline{\frac{du}{dx} = e^x},\ \underline{\frac{dx}{du} = \frac{1}{e^x}},\ \underline{\frac{dx}{du} = \frac{1}{u-1}}\right\}$	Differentiating to find any one of the <u>three underlined</u>	<u>B1</u>
	$= \int \frac{e^{2x}.e^x}{1+e^x}\,dx = \int \frac{(u-1)^2.e^x}{u}.\frac{1}{e^x}\,du$ or $= \int \frac{(u-1)^3}{u}.\frac{1}{(u-1)}\,du$	Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1+e^x$ or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1+e^x$.	M1*
	$= \int \frac{(u-1)^2}{u}\,du$	$\underline{\int \frac{(u-1)^2}{u}\,du}$	A1
	$= \int \frac{u^2-2u+1}{u}\,du$ $= \int u - 2 + \frac{1}{u}\,du$	An attempt to multiply out their numerator to give at least three terms and divide through each term by u	dM1*
	$= \frac{u^2}{2} - 2u + \ln u\ \ (+c)$	Correct integration with/without +c	A1
	$= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$	Substitutes $u = 1+e^x$ back into their integrated expression with at least two terms.	dM1*
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$		
	$= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ **AG**	$\underline{\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k}$ must use a $+c$ and "$-\frac{3}{2}$" combined.	A1 **cso**
			(7)
			[13]

Question Number	Scheme		Marks
7 (a)	At A, $x=-1+8=7$ & $y=(-1)^2=1 \Rightarrow A(7,1)$	$A(7,1)$	B1 (1)
(b)	$x=t^3-8t$, $y=t^2$,		
	$\frac{\mathrm{d}x}{\mathrm{d}t}=3t^2-8$, $\frac{\mathrm{d}y}{\mathrm{d}t}=2t$		
	$\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2t}{3t^2-8}$	Their $\frac{\mathrm{dy}}{\mathrm{d}t}$ divided by their $\frac{\mathrm{dx}}{\mathrm{d}t}$ Correct $\frac{\mathrm{dy}}{\mathrm{d}x}$	M1 A1
	At A, $\mathrm{m}(\mathbf{T})=\underline{\frac{2(-1)}{3(-1)^2-8}}=\underline{\frac{-2}{3-8}}=\underline{\frac{-2}{-5}}=\underline{\frac{2}{5}}$	Substitutes for t to give any of the four underlined oe:	
	$\mathbf{T}$: $y-(\text{their } 1)=m_T\left(x-(\text{their } 7)\right)$ or $1=\frac{2}{5}(7)+c \Rightarrow c=1-\frac{14}{5}=-\frac{9}{5}$ Hence $\mathbf{T}$: $y=\frac{2}{5}x-\frac{9}{5}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y=(\text{their gradient})x+"c"$.	dM1
	gives $\mathbf{T}$: $\underline{2x-5y-9=0}$ **AG**	$\underline{2x-5y-9=0}$	A1 **cso** (5)
(c)	$2(t^3-8t)-5t^2-9=0$	Substitution of both $x=t^3-8t$ and $y=t^2$ into $\mathbf{T}$	M1
	$2t^3-5t^2-16t-9=0$		
	$(t+1)\{(2t^2-7t-9)=0\}$ $(t+1)\{(t+1)(2t-9)=0\}$	A realisation that $(t+1)$ is a factor.	dM1
	$\{t=-1 \text{ (at } A)\}$ $t=\frac{9}{2}$ at B	$t=\frac{9}{2}$	A1
	$x=\left(\frac{9}{2}\right)^2-8\left(\frac{9}{2}\right)=\frac{729}{8}-36=\frac{441}{8}$ = 55.125 or awrt 55.1 $y=\left(\frac{9}{2}\right)^2=\frac{81}{4}=20.25$ or awrt 20.3 Hence $B\left(\frac{441}{8},\frac{81}{4}\right)$	Candidate uses their value of t to find either the x or y coordinate One of either x or y correct. Both x and y correct. awrt	ddM1 A1 A1 (6)
			[12]